Common Core Math in High School

We will discuss all of high school together, as opposed to one class (Algebra 1 or Integrated Math 2) or one grade at a time. Regardless of names and labels, high school math should primarily be about modeling with algebra and functions. Our discussion here includes examples, which at high school can be sophisticated. Even if you don't follow all of the math, we hope you get a sense for how things are thought through and are designed to better serve your student. Here are some suggestions for what concerned parents might do to help their kids, followed by a discussion of the major shifts in Common Core high school math.

Tips for parents:

- Talk about career choices, and investigate together what math is required for a university or associate degree, a technical certificate, or possible on-the-job needs. Plenty of jobs use math, especially things like proportional reasoning and linear functions, jobs ranging from nursing to forestry to operations to accounting to computer-aided design to carpentry.

- Make sure your kids understand fractions and middle school math — especially proportional reasoning — super well. Work on real-world problems in daily life to reinforce these skills. For example, you can discuss financing their college, or have them imagine what their budget will be when they are 25 years old and discuss financing a car. Too often, in the past, arithmetic skills were lost (because they were based on memorization and hadn't been reinforced) by the time kids got to college.

- Consider using resources such as Mathalicious or Dan Meyer 3-Act Tasks if you want enrichment or extra practice. Khan Academy is working on Common Core skill practice as well.

- Enjoy math! It feels good to put some effort forward and figure something out. Work on your own to model this. Google "Carol Dweck mindset" to understand how important attitudes towards effort and learning are.

Common Core Shift: Applied problems, often based on simpler math

Most math in the world is done to serve some application to science, business or daily life. But we have not taught application of math well! A famous study from the '80s with freshman engineering students showed they had essentially no skill in even setting up equations based on simple situations like "there are six students for each professor."

A look at our old textbooks provides a good explanation: we haven't actually taught applied math. We give students "word problems" exactly like those already worked out in the text. But in real life, when we need to understand a financing plan, no one tells us "look at page 314 of your book to see how to do a very similar problem." We have been denying our kids the opportunity to use the math like we want them to! A TED talk by Dan Meyer (http://www.ted.com/talks/dan_meyer_math_curriculum_makeover) gives one teacher's terrific explanation of how our old textbooks have actually hurt kids' problem solving abilities.

Consider an example with cell phone plans:

Plan 1: $50/month with unlimited voice and data
Plan 2: $25/month with $0.10 per minute or MB of data

We can "just figure this out" — the $25 difference up front would pay for 250 minutes of talking or MB of data. So if we think we'll use less than that 250 minutes/MB then the second plan is better, and if we use more the first plan is better.

Now suppose there are three plans:

Plan 1: \$50/month with unlimited voice and data
Plan 2: \$25/month with \$0.10 per minute or MB of data
Plan 3: \$35/month with \$0.05 per minute or MB of data

"Just figuring it out" becomes complicated. This is the kind of problem where more systematic approaches supported by math are important (see solutions at http://www.illustrativemathematics.org/illustrations/472). Things are better organized using graphs, which show how to find the cheapest plans across different possible minutes/ megabytes.

Interestingly, this task is grounded in eighth grade math, which is when students will first study linear functions. These plans are all perfectly represented by linear functions, since the per-minute rates are constant (in one case zero). In the classroom, students might see this kind of problem first as a whole-class or group project at eighth or ninth grade. They should in later grades be given chances to do these kinds of problems more on their own. In eleventh grade, on the Smarter Balanced assessment, this is the kind of activity that can be part of a "performance task."

By asking students to do these kinds of tasks, we are saying that in addition to some more advanced high school math, authentic application of simple math is important for college and career readiness. Research papers in disciplines like economics often use exactly this kind of "eighth grade" math. More sophisticated math can arise from questions as simple as "why are honeycombs hexagonal?"

Some of the most successful high school teachers have been using these kinds of activities well before the Common Core. Dan Meyer (Google "Dan Meyer three-act tasks"), Fawn Nguyen, a group of teachers with a website called "Mathalicious" (which has some free and some fee-based lessons), and the Mathematics Assessment Project provide many examples. Mathalicious developed an activity called "Text Me Later," for example, in which students time each other texting short messages, and then calculate how far a car travels in that time. These kinds of activities lend themselves to project-based learning, team teaching, and community engagement.

Common Core Shift: Purpose for math skills

A second change that the Common Core prescribes is for skills, especially in algebra, to be applied with a purpose in mind. Not only should skills be applied fluently, but students should recognize *when* and *why* they should be applied. For example, in algebra students have long been asked to simplify quadratic expressions or solve quadratic equations. Now, students may be given the height of a rocket as a function of time,

$h(t) = -16(t^2 - 2t - 5)$ feet

and asked to put it in completed square and factored form as shown below.

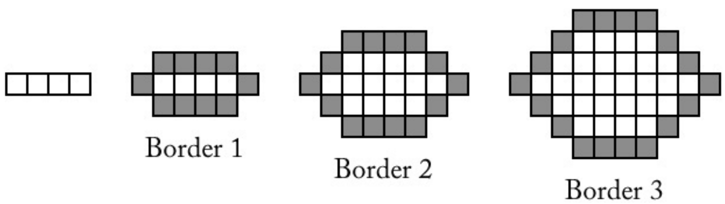
$h(t) = -16(t-1)^2 + 96 \qquad h(t) = -16\left(t - \left(1 - \sqrt{6}\right)\right)\left(t - \left(1 + \sqrt{6}\right)\right)$

Rather than just an arbitrary skill, this reveals its maximum height and total time of flight. The initial equation can be used to see that when $t = 0$ then the height is 80 feet, meaning the rocket was launched from that high (the top of a building?). The second form is a negative (or zero) number added to 96, which means the maximum height is 96, happening t – 1 = 0 or one second later. The third form shows the height as zero at two possible times, when each factor is zero. Only one of those is positive, namely $t = 1 + \sqrt{6}$ or a bit over 3 seconds later, so that is when the rocket hits the ground. The algebra is similar to what has been asked in the past, converting between different forms of the same function — but just as different forms of fractions are useful for different purposes, the same can be said about different algebraic forms. Switching between algebraic forms is enough work that most people would want to see some payoff from that work. In this case, that means being able to measure the rocket as it is launched and know without further measurements how high it will go and how long it will be in the air.

Purposeful constructions also occur in geometry. In this task (http://www.illustrativemathematics.org/illustrations/508) students are asked to place a fire hydrant at an equal distance from three locations. In class discussion, students can see the purpose for compass and straight-edge constructions which seem arbitrary (and a mouthful to say), like the perpendicular bisectors used here. A teacher gives insight about her experience with the task here: http://easingthehurrysyndrome.wordpress.com/2013/10/14/placing-a-fire-hydrant-2/

In the above two examples, the purpose for some mathematical skills came from a "real-world" context, but purpose is broader than that (so different from our previous discussion). Purposes can include supporting one's own idea about an interesting problem, refuting an alternative idea, or giving a clear mathematical description as in the following task (http://www.illustrativemathematics.org/illustrations/215):

Fred has some colored kitchen floor tiles and wants to choose a pattern using them to make a border around white tiles. He generates patterns by starting with a row of four white tiles. He surrounds these four tiles with a border of colored tiles (Border 1). The design continues as shown below:



Fred writes the expression $4(b-1)+10$ for the number of tiles in each border, where b is the border number, $b \geq 1$.

1. Explain why Fred's expression is correct.
2. Emma wants to start with five tiles in a row. She reasons, "Fred started with four tiles and his expression was $4(b-1)+10$. So if I start with five tiles, the expression will be $5(b-1)+10$." Is Emma's statement correct? Explain your reasoning.
3. If Emma starts with a row of n tiles, what should the expression be?

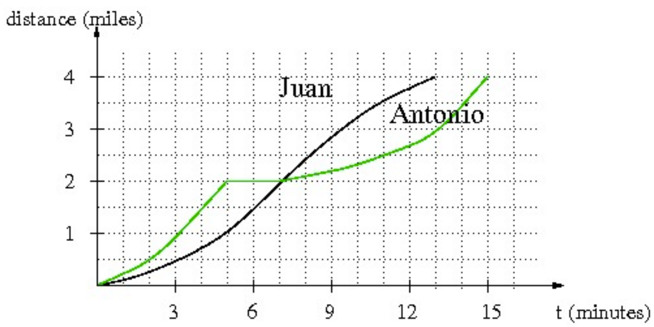
Common Core Shift: Developing meaning, to promote better mathematical skills and application

With new and sometimes greater expectations for what we want students to be able to do, a key question is: "How are we going to help students get there?" One main answer is that we're going to promote full understanding of what they do. Meaning and methods together are the foundation for mastery such as being able to do real-world problems you haven't seen exactly before.

This will start in early grades, for example placing fractions on the number line to understand how they add, as opposed to only memorizing steps to add them. But even in the transition to the Common Core, students can start making sense of mathematics at any time. We've had plenty of positive experience in helping college students understand their elementary math better!

These meanings are often tied in with essential life skills, such as interpreting graphical information. Consider the following task, https://www.illustrativemathematics.org/illustrations/633:

Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).



1. Who wins the race? How do you know?
2. Imagine you were watching the race and had to announce it over the radio. Write a little story describing the race.

While a great beginning task, it can be a bit tricky even for those who read graphs all the time, since at first it looks like "Antonio goes further." Also, it is a writing task! What a wonderful opportunity for a math teacher and a language arts teacher to team-teach. Finally, think about how we have prepared students to read graphical information in the past — through tasks such as "graph $x^2 - 3x + 4$?" Those have their importance, but it is clear we have been missing tasks like this one that requires students to interpret the information in the graph rather than simply manipulate it.

In the past students were taught to rattle off the basic equation for a line as

$$y = mx + b$$

But what does a linear function *mean*? It means that for every unit one quantity changes, another changes by a fixed amount. For example, for each rubber band added, Barbie is a fixed amount closer to the floor in this activity: http://fawnnguyen.com/barbie-bungee/. Yes, linear functions are given by $y = mx + b$, but that's just part of the meaning. Students need to connect this part of the meaning with other parts (for example, the m in the Barbie activity is the further amount she falls when one rubber band is added) in order to be able to actually use linear functions like we want them to.

What if, instead of adding some fixed amount each time, a quantity gets multiplied each time? That's what an exponential function *means*. Exponential functions have more importance in the Common Core, often appearing earlier than they have in standard curricula because they have a more basic meaning and thus appear in more applications than polynomial functions (traditionally introduced earlier).

Even more basic is the concept of a function and use of function notation, which is at the top of the list — along with some facility with algebra — of what professors need students to know coming into college-level math classes. It takes a lot of time to fully develop the meaning of functions, which can be defined through expressions such as $x^2 + 2$, or 1.2^x, but also as graphs, as tables of (representative) values, as stories (like in the bike task above), or as processes (the value increases by 5 each time). The time taken to understand functions in different ways will pay dividends when students must learn skills such as adding or composing functions. Just as students with better "number sense" will use arithmetic more effectively, students who really understand that linear growth is about constant rates of change will have more "function sense" and be more likely to apply what they know about functions to daily life, college and careers.