Understanding Linear Measure
Push your instruction beyond procedures: Enhance student tasks and offer better opportunities to develop conceptual understanding.

By Leslie C. Dietiker, Funda Gonulates, and John P. Smith III

If you teach mathematics to students in kindergarten through grade 4, what do your students know about linear measurement? For example, consider a grade 1 assessment task from Everyday Mathematics (see fig. 1). How do you think your students would respond? What might you learn about their understanding of length measurement? What might you not learn? Think about how you would respond to this task before reading further.

Although the Curriculum Principle in Principles and Standards for School Mathematics states, “A school mathematics curriculum is a strong determinant of what students have an opportunity to learn and what they do learn” (NCTM 2000, p. 14), this article seeks to help teachers enhance tasks found in commonly used textbooks to create additional opportunities for students to develop conceptual understanding of linear measurement specifically involving unit iteration. Guided by the work of the STEM (Strengthening Tomorrow’s Education in Measurement) Project, which carefully analyzed the measurement content found in three representative U.S. textbooks, we also raise questions about opportunities that are often absent from elementary school mathematics curricula.
The research on curricula and student knowledge

Studies have shown that elementary school students’ understanding of linear measurement, specifically concerning unit iteration, is surprisingly weak. For example, results from the 2003 National Assessment of Educational Progress (NAEP) in mathematics showed that only 20 percent of fourth graders correctly identified the length of a toothpick as 2.5 inches (see fig. 2). Other answers for this open-response task, referred to in this article as the Toothpick task, include 10.5 inches and 3.5 inches (14 percent and 23 percent of responses, respectively).

One way to understand why the length of the toothpick is 2.5 inches is to apply the principle of unit iteration (see fig. 3). Unit iteration is the act of identifying a unit (in this case, an inch), moving it along the length of the object to be measured without gaps or overlaps, and counting the number of units required. In contrast, filling a span with a collection of equal-size units is commonly called tiling.

In A Research Companion to Principles and Standards for School Mathematics (2003), Lehrer argues that both unit iteration and tiling are fundamental aspects of understanding length measure. Both require the attribute of the object being measured (the length of the toothpick) to be exhausted, without gaps between units. Additionally, the units must lie along the path of the dimension (such as the straight path connecting the ends of the toothpick). In tiling situations, students must also recognize the need for equal-size units (whereas in iteration, only one unit is used). Therefore, unit iteration and tiling both satisfy four basic requirements: The units must—

- be placed on the path being measured;
- have equal length;
- exhaust the quantity being measured without gaps; and
- not overlap; otherwise, part of the quantity would be measured more than once.

Although it may seem trivial, another key to understanding the Toothpick task is to recognize that a ruler represents tiled units of length (in this case, an inch or a quarter of an inch). If students view a ruler as only a tool with marks rather than as a length partitioned into equal-size units, they may be able to use the ruler correctly only when they can align the zero mark with one end of the object (which would explain the response of 10.5 inches). Also, confusion about what the marks on rulers represent may explain the response of 3.5 inches (which could be produced by counting inch tick marks along the toothpick and adding one-half of an inch for the remainder).

Results on a task similar to the Toothpick task led Kamii (2006) to question why instruction for length measurement has been ineffective. She noted that students are usually asked to produce a measure number and are not often asked to compare measures. For example, students can move a yardstick across the chalkboard and generate a number without focusing on the conceptual principles of measurement that are involved. Kamii suggests that having students make their own rulers or yardsticks might present opportunities for them to understand how rulers represent tiled units of equal length.

Kamii’s claim that teaching measurement usually focuses on carrying out procedures is supported by our STEM Project findings (Smith III et al. 2008). STEM analyzed three U.S. elementary school mathematics curricula to learn how they support students’ learning of measurement. These three were chosen to represent some of a wide range in mathematics curricula at the elementary school level; the first two have large shares of the textbook market (Reys and Reys 2006). All three curricula (Everyday Mathematics [EM], Scott Foresman-Addison Wesley’s Mathematics [SFAW], and Saxon Math [Saxon]), have a high proportion of procedural content (focusing on the processes of measuring and estimating spatial quantities) and relatively little conceptual content (the basic principles that underlie and justify measurement procedures). The analysis of these three textbook series suggests that many U.S. elementary school teachers are using written materials in which less than 10 percent of measurement content clearly and explicitly identifies—or asks students to articulate—basic conceptual principles. And when conceptual principles are expressed, references to unit iteration are infrequent.

To better understand what kind of unit iteration is found in U.S. texts, the STEM Project identified any conceptual and procedural content in these textbooks involving unit iteration or
tiling. With regard to conceptual development, we noted any statements, questions, or problems focused on understanding the principles of unit iteration (iterate unit) or how units are represented on a ruler (ruler represents iterated unit). The analysis suggests that U.S. elementary school texts contain very little conceptual development of unit iteration in K–grade 3 and that content involving how a ruler represents iterated units of length is particularly rare. For example, with regard to rulers representing an iterated unit of length, SFAW and Saxon contained only one instance across K–grade 3, and EM contained only three.

**Revisiting the tasks**

Because we found conceptual content sparse in the textbooks we examined, we focused on how teachers can analyze existing curricular tasks and modify them to increase students’ conceptual understanding of linear measurement. When analyzing a task, consider the opportunities that specific tasks may offer students to learn about unit iteration; also consider the opportunities they do not explicitly offer. Next we analyze three tasks found in two of the textbooks. We selected these tasks because they offer some attention to conceptual understanding of measurement and yet can still be enhanced to include understanding of iteration as described in the previous section. In the next section, we suggest how to analyze and adjust any measurement task to increase student opportunities to understand linear measurement.

**The Buttons task**

A task that prompts students to consider four different tilings, the Buttons task (see fig. 1) has students decide which fictional student “made the best measurement.” When responding to “explain your answer,” a student may state why his or her selection represents the best measurement and describe the principles of measurement on which it relies. The different tilings give students the chance to attend to the attribute being measured (e.g., Is it the distance across the page? Is it the diagonal distance?), whether the units must be placed in a straight line, and whether the units may have gaps. Of 1653 textbook pages that contained linear measurement, the toothpick task is an open-response NAEP test item for grade 4.

**The Buttons task has value, but considering how weak elementary school students’ understanding of unit iteration is, this task could easily be enhanced to improve their grasp of measurement concepts.**

**Measuring the Page**

Here is how four children used buttons to measure the width of this page.

![Image of buttons and rulers](https://example.com/buttons.png)

Who do you think made the best measurement? Explain your answer.

---

**The Toothpick task is an open-response NAEP test item for grade 4.**

What is the length of the toothpick?

![Image of toothpick on ruler](https://example.com/toothpick.png)

Unit iteration is one way to find the length of a toothpick.

![Image of toothpick measurement](https://example.com/measurement.png)
content, this task was the only instance we found that explicitly prompted students to consider (1) the consequences of gaps and (2) the need for the units to lie on the path being measured (i.e., the straight line across the page).

Despite these beneficial qualities, the Button task does not require students to consider what would happen if the tilings had used different-size buttons. Moreover, students can answer correctly just by considering Rodrigo’s measurement first, without seriously considering the work of other students. Because the task asks students for only the “best measurement,” they are not required to describe the principles of unit iteration that the other tilings violate, such as the fact that Josef’s units do not exhaust the width of the page. So, although this task has value, it could be enhanced if a fifth fictitious student who used different-size buttons were added and if students were instead asked to explain to the other fictitious students (those without the best measurement) how to improve their measurements.

The Foot task
In another task from EM, students are given only one length unit (a tracing of their foot) and asked to measure a span, such as the length of a table. The Foot task requires students to iterate their single foot cutout to measure length. If the span is not marked with a line (such as floor tiles), students must also choose a path that will produce an accurate measurement (see fig. 4).

Although this task asks students to complete a measurement procedure, it also offers opportunities for conceptual development. For example, if students iterate with a “finger gap” between successive units, you might bring attention to the gap: "Is this part of the length being measured? Why, or why not?” Also, if you notice that students measure a straight attribute (such as the width of a rectangular table) with a curved path or one that does not span the entire length of the object, ask, “If I had a string of [the faulty length measured by the students] and stretched it straight across the table, would it be the same length? Why, or why not?” This task does not require students to address the need for the units spanning the object to be the same length, but you could enhance the task if you propose measuring the length by tiling with all the students’ cutouts and ask, “What if I used everyone’s cutouts? Would that give us an accurate measure? Why not?”

The One-Inch Squares task
The third task we highlight is one of the few we found that explicitly tries to help students recognize that rulers represent iterated units of length. This task prompts the teacher to demonstrate how to measure an object by tiling one-inch squares. Then the teacher is directed to demonstrate finding the length of the same object using a ruler and to ask, “Why do you get the same answer?” (see fig. 5).

However, although students may concretely recognize each square tile as a distinct unit along the span, whether this is enough to help students recognize discrete one-inch spans on the ruler is unclear. Therefore, you could enhance this task by having students also lay a ruler along the tiled one-inch squares and count the units as shown on the ruler. In addition, you could demonstrate that if the ruler is not placed so that the zero mark aligns with the end of the object, the length of the object (as shown by the tiled one-inch squares) does not change. This way, students can begin to recognize that the one-inch units on the ruler are between the marks and that it is the number of these spans along the object that matters.

Building conceptual understanding
The three tasks we have discussed offer students opportunities to develop conceptual understanding of unit iteration because they explicitly address principles of linear measurement. But if these opportunities are relatively rare in textbooks, what can a teacher do? One option is to enhance the existing tasks. Another is to change the discourse around a task. In this section, we offer four questions to identify missed opportunities for conceptual development. We suggest how to enhance the task as well as some questions the teacher could use for discussion when the answer to any of these questions is no.

1. Does the task offer the opportunity for students to think about whether units need to have equal length?

   A typical textbook task asks students to measure object lengths with nonstandard units. Consider supplying multiple manipulatives as length units and asking a question
such as, “I like how the red button and the pink erasers look together; can I use them both to measure the length at the same time? Why, or why not?” Some students may not realize that to ensure units are the same length, they must also be rigid. Consider furnishing a rubber band for students to measure a length. Ask, “Could I use this to get an accurate measure of length? Why, or why not?”

2. Does the task offer an opportunity for students to think about why units cannot have gaps or overlaps?

When a task prompts students to tile units, consider including an erroneous example on the board (such as Josef’s work in the Buttons task) and asking students to explain why such a method will not give an accurate measurement. When a task prompts students to iterate a unit of length (such as in the Foot task), consider having them trace the units before moving them, so they leave a record of where the units were placed along the span.

The Foot task offers the chance for conceptual development.

Tasks like One-Inch Squares give students a chance to link their work with discrete length units (nonstandard and standard) to the marks on rulers.

---

Figure 4


Figure 5

This will not only help students “see” prior units that might disappear without visual evidence but will also allow you to spot a gap and ask, “Is this portion being measured?” You might also focus on the need for units to lie on the path being measured, with such comments as, “I like how you placed your units in a straight line. Why do they need to be placed in a straight line? What if the line curved instead?”

3. Does the task offer an opportunity for students to think carefully about what quantity is being measured?

Students are often prompted to find the length of objects in the classroom. If a task asks students to measure an attribute of an object, such as the height of their desk or the width of an eraser, leave the attribute of the object ambiguous by not pointing to it or marking what part to measure. For example, the width of a table could be interpreted in multiple ways, causing groups of students to get different measures. If this happens, reports from different groups could lead to a discussion about what specific attribute students saw as the “width,” which could, in turn, help students learn that sometimes saying words like width or length does not clearly communicate the attribute they are measuring.

4. Does the task offer an opportunity for students to think about how rulers represent iterated units of length?

All three elementary school curricula offer ample opportunities for students to measure with a ruler. Ask, “What if I place the ruler so that the left end of the object is aligned at two inches instead?” to spur ideas about how to use the ruler to measure length in this way. Also ask students to think about why the object does not change length even when the other end aligns with a new number. Another strategy is to present some “strange” rulers (see fig. 6), and ask, “Could I use these to measure correctly? Why, or why not?”

**Conclusion**

Although measurement is among the most meaningful topics in the elementary school mathematics curriculum, evidence from diverse sources suggests that many students are not learning measurement well. Many schoolchildren are learning procedures for measuring length (how to place and read a ruler in the standard way) without learning the underlying conceptual principles. This deficit makes errors more likely, especially when students must adjust a standard procedure, such as measuring with a broken ruler that is missing the zero mark.

One way teachers can address this challenge is to look for tasks that hold promise for raising and clarifying conceptual issues and then enhance their usefulness without lots of work. We should not expect “good” tasks to address all four key issues presented in this article. No one task will teach all the fundamental principles of linear measurement. Instead, we offer this frame and our assessment of three tasks as tools for teachers to examine, reflect on, and use to improve their measurement curriculum.

**REFERENCES**


Development and Publication of Elementary Mathematics Textbooks: Let the Buyer Beware!”  

The authors gratefully acknowledge the rest of the STEM Project team for its hard work and comments on this article; Janet Prybys for her helpful input; and the National Science Foundation (NSF) REESE Program (REC-0634043) in developing these ideas—whose content is our own and does not express the views of the NSF. We are grateful to Scott-Foresman/Addison Wesley and McGraw-Hill for permission to reprint textbook excerpts.

Leslie C. Dietiker, dietike4@msu.edu, is a doctoral student in the Division of Science and Mathematics Education at Michigan State University (MSU). As a textbook author, she is interested in improving written curriculum to increase student interest in math and access to it. Funda Gonulates, gonulate@msu.edu, also a doctoral student in the Division of Science and Mathematics Education at MSU, is interested in teachers learning from written curriculum materials; their development of curriculum; and the connections within and outside the mathematics, specifically those between algebra and geometry. John P. (Jack) Smith III, jsmith@msu.edu, is an associate professor in the Department of Counseling, Educational Psychology, and Special Education and a member of the Mathematics Education faculty at MSU. He focuses work on student learning and understanding of math and the factors that shape those processes.

---

**NCTM’s 2011 Member Referral Program**

**Let’s Add Up! Refer. Receive.**

Participating in NCTM’s Member Referral Program is fun, easy, and rewarding. All you have to do is refer colleagues, prospective teachers, friends, and others for membership. Then as our numbers go up, watch your rewards add up.

Learn more about the program, the gifts, and easy ways to encourage your colleagues to join NCTM at [www.nctm.org/referral](http://www.nctm.org/referral). Help others learn of the many benefits of an NCTM membership—Get started today!