Connecting stories to qualitative coordinate graphs has been suggested as an effective instructional strategy (Blubaugh and Emmons 1999; Maus 2005; NCTM 2000). Even students who are able to create bar graphs may struggle to correctly interpret them. Giving children opportunities to work with qualitative graphs can help them develop the skills to interpret, describe, and compare information from a graph even without the availability of numeric labels. This investigation addresses the Data Analysis and Probability Standard (NCTM 2000) and explores the value of connecting stories with qualitative bar graph instruction, which too often focuses on only counting, tallying, and creating bar graphs.

Eighteen students in a diverse second-grade class at the Days Park Bilingual Early Childhood Center in Buffalo, New York, could construct bar graphs by counting and tallying objects, but they had no experience with qualitative graphs. They explored properties of both quantitative bar graphs (i.e., graphs with numeric labels) and qualitative bar graphs (i.e., those without numeric labels) by creating and identifying “mystery” bar graphs. Their culminating activities of matching graphs to nursery rhymes, songs, and stories entailed an additional two days.

Investigation objectives
By the end of their exploration, students should be able to demonstrate the following skills:

- Write descriptive statements about bar graphs.
- Understand that different bar graphs may share the same mathematical features.
- Interpret and label qualitative graphs in a meaningful way.
- Connect qualitative graphs with contexts of nursery rhymes, songs, and stories.

Materials

Lesson 1 (day 1)
Each student needs the following items:

- Crayons
- 3 stacks of 5 connected Unifix® cubes (at least 1 blue, 1 green, and 1 yellow stack)
- Activity sheet 1

Lesson 2 (days 2–3)
Each student needs the following items:

- Crayons
- 4 stacks of 5 connected Unifix cubes (at least 1 blue, 1 green, 1 yellow, and 1 red stack)
- A four-color cube graph (see p.432)
- Activity sheet 1 (see full-sized versions online)
- Activity sheet 2
- Activity sheet 3 (several copies)

Lesson 1 investigation
On the first day, students investigate and gain experience with bar graphs that are not labeled with numbers. To begin, give each student three stacks of five connected Unifix cubes (one blue stack, one green, and one yellow) and a copy of activity sheet 1. Ask students to choose some cubes, set the rest aside, and then use crayons to create a bar graph of their selected cubes on the activity sheet.
Most of our second graders created graphs without using numbers, but five of the eighteen wrote some sort of number sentence (see [Fig. 1]) or number label on the graph. Francisco even numbered each cube, thus creating a vertical axis for each bar.

Next, have students write a sentence about their graph. Exactly half our second graders referenced a specific number of cubes in their sentences, samples of which follow:

“My graph looks like steps, and it tells my Mom’s and my favorite colors.”

“I put my graph in number order.”

“Blue was my most favorite. My graph went down from five to three. I have twelve cubes in all.”

“My graph is almost all the same because I have 3 yellows and 3 green.”

“I ate 5 cookies; then I ate 1 Oreo; then I ate 3 strawberries; and all of it was good.”

[This number story described a graph of five blue, one green, and three yellow cubes.]

The next part of the activity encourages students to describe graphs without using numbers. Avoid voicing specific numbers and make a statement such as, “Stand if your graph has more blue cubes than any other color.” Have students stand and display their graph if it meets the conditions of the statement. At this point, it is important to discuss that many graphs may meet a single description and still all be different from one another. The two graphs in [Fig. 1] are examples with more blue cubes. You may want to point out that if a graph meets the given criteria (i.e., “more blue than any other color”), its other features are irrelevant.

Once students understand the open-ended nature of this activity, have each child propose a description to the class and have the others
stand and display their graph if it matches the description. About half our students used numbers in their descriptions. Typical descriptions included the following:

“Green and yellow are the same.”
“The most are green.”
“Three yellow”
“Only one of a color”
“Less green than yellow”
“None of the bars are the same height.”
“Low blue, low green, low yellow”
“Four blue and five green”

These descriptions presented several opportunities for rich discussions about mathematical language. For example, when “more than twelve altogether” was posed, Wandalee asked if she should stand up because she had exactly twelve cubes in her graph. Her classmates referred to a word wall, named some examples of “more than” and “not more than,” and then—on their own—agreed that she should not stand up.

A startled silence ensued when Destiny presented “no cubes” as her description. After some false starts and thinking time, there was a rush as all the students quickly sat down. Many began laughing. When they were asked what a graph with no cubes looks like, they correctly identified it as both an empty graph and a graph with zero cubes for each color.

Lesson 2 investigation
The next activities focus on interpreting qualitative graphs and determining number values that they could reasonably represent. Give each student four stacks of five connected Unifix cubes (one blue stack, one green, one yellow, and one red) and a fresh copy of activity sheet 1.

Begin by describing one or more features of a bar graph, such as, “There are fewer green cubes than yellow cubes.” Have students use their cubes to create a graph matching your description by laying selected cubes on top of the activity sheet. Ask several students to share their graph and explain how they matched the given criteria. Emphasize that many different graphs can correctly match the given description.

Continue the activity by having some students pose criteria and others respond by creating graphs that match. Najya’s description of “zero blue, five green, and zero yellow” drew giggles, but the entire class correctly displayed only five green cubes.

Show a four-color bar graph with no scale or grid. Ask students to share a sentence about the graph and explain how they know the statement is true when no numbers appear on the graph. Shown figure 2a, our class offered these:

“They are all different heights.”
“Green is the tallest.”
“Red is the smallest.”
“Blue and yellow are the same.”
“Yellow is taller than red.”
When prompted, students correctly expressed their ideas in language that showed they recognized that each bar in the graph represented a specific number of cubes. For example, the fourth comment above was clarified as, “There are the same number of blues and yellows.” However, even after repeating this process with ten different graphs, most students continued to use language describing the bars’ visual appearance (such as taller, smaller, and shortest), rather than language referring to the number of cubes (such as more than, less than, or fewest).

The graph in figure 2b led to a productive mathematical discussion when—for the first time—a student described the graph in terms of specific numbers that could be the bar heights, responding, “One, two, three, four.” We asked if the graph could represent other numbers, and Gianni offered, “Ten, twenty, thirty, forty.” Although the class was quite fluent in counting by twos and by fives, no one offered any other possible numeric values for the graph. To push their thinking, we asked if the bar heights could be one, twenty, twenty-one, and fifty. The class quickly agreed that could not be correct because “they would have to be the same,” meaning the increase from bar to bar is constant, so the increase in numbers would also have to be constant.

Give each student a copy of a four-color bar graph that you have created. The graph does not need to be exactly like either of the samples in figure 2, but the height of each bar should be exactly equal to the height of a stack of the type of cubes that your students are using.

We asked students how they could determine the number of cubes for each color if there were no numbers, and they quickly suggested using their cubes to measure. Almost all of them measured a bar’s height with only the corresponding color of cube. That is, they measured the blue bar with only blue cubes. Not surprisingly, about two-thirds of the class was stumped with the yellow bar on one of our handouts, because it was ten cubes tall and they had only five yellow cubes. A series of increasingly specific scaffolding prompts (Can you think of a way to see how tall this is? Are the different colors of cubes the same size? Do you have to measure with yellow only?) gave all students the opportunity to successfully develop a strategy to measure the yellow bar. Some used the yellow stack of five cubes, made a mark, and then used the yellow stack again to measure the remainder of the bar. Others used the yellow stack and then measured the rest of the bar with cubes of a different color.

Give your students a copy of activity sheet 2, and ask them to create and color their own mystery cube graph without numbers. Emphasize these two points:

1. Make the height of each bar equal an exact number of cubes because you will use your cubes to measure the heights.
2. Do not write any numbers on the graph or you will spoil the mystery.

Some of our students simply colored their bars (see fig. 3a); some marked the top of each bar; and others outlined each individual cube. When a student’s bar heights did not represent the heights of actual cubes, we had that student try to measure the graph with cubes and then adjust the heights before proceeding.

After your students complete their mystery graphs, give each one several copies of activity sheet 3. Have classmates exchange the graphs that they created on activity sheet 2 and use
cubes to determine how many cubes each bar on the graph represents. They can use one copy of activity sheet 3 to record their findings for each mystery graph that they measure. Our students stayed engaged in this activity and determined almost all the heights correctly, even for bars taller than five cubes. We noticed that some students continued to measure bars using only the corresponding color of cube, but others began to think more abstractly and use the same stack for all the bars, regardless of color.

After the second graders at Days Park Bilingual Early Childhood Center had explored properties of both quantitative and qualitative bar graphs and had created and identified “mystery” bar graphs, their culminating activities were to match graphs to nursery rhymes, songs, and stories. Lesson 3 entails an additional two days.

Figures 6–10 accompany the online version of this article at www.nctm.org/tcm.

Lesson 3 (days 4 and 5)
Each student needs the following items:

- **Large** versions of unlabeled graphs (see figs. 4a, 5a, 6a, 7, 8, 9, 10)
- **Copies** of your choice of qualitative bar graphs and corresponding stories, songs, or poems
- **Rebus-style** nursery rhymes (optional)

**Lesson 3 investigation**

Have students read and sing nursery rhymes, poems, songs, and books; match them to qualitative bar graphs; and label and title the graphs appropriately. For example, on the board we displayed a large version of the graph in figure 4a. Students began by reading the following version of the nursery rhyme “Baa, Baa, Black Sheep”:

Baa, baa, black sheep, have you any wool?  
Yes, sir; yes, sir; three bags full.  
One for the master, one for the dame, and  
One for the little boy who lives down the lane.

We asked how the graph could be used to represent the nursery rhyme. After some discussion, we noted that the bars are all the same height and each person in the nursery rhyme gets the same number of bags of wool. So, it made sense to students to name the vertical axis bags of wool and label it with the numeral 1 at the appropriate height. Deciding how to label the horizontal axis took more prompting, but once students decided on the man for the first bar, they quickly produced a finished graph (see fig. 4b).

Students received a rebus-style version of “This Old Man,” which the class sang together.
We displayed three more oversized graphs on the board (see fig. 5a; see fig. 6a online). When asked which of the three could match “This Old Man,” students immediately choose correctly. They quickly named the graph and labeled the bars. However, they again had difficulty titling the axes. After much discussion, they named the $x$-axis as things playing (referring to the object that the old man plays in that verse) and the $y$-axis as number he is playing (i.e., the verse number).

The class ended this part of the lesson by singing from rebus-style versions of “Five Little Speckled Frogs” and “Five Little Ducks” and then identifying and completing both graphs (see fig. 6b online).

The youngsters spent the remainder of the investigation working in groups, reading books and poems, matching them to qualitative graphs, and completing the graph information. Although these are qualitative graphs, it is important that the relationships among the bar heights is accurate. For example, if one bar is five units and the next is ten, the second bar must be twice as high as the first.

Typical counting stories, songs, and poems, such as “The Twelve Days of Christmas,” increase by ones (see fig. 7 online), but many others increase by other units, decrease, both increase and decrease, or change units during the story (see figs. 8, 9, and 10 online). In almost all cases, after our students chose the graph to represent a particular story, they also created a numbered vertical axis to help them correctly complete the labeling.

Beyond the lesson
This investigation focuses primarily on using qualitative graphs to deepen students’ understanding of bar graphs. However, in addition to developing such mathematical language as less, more, fewer, and so on, our students also made many connections to literature and reading comprehension.

To extend the investigation even further or challenge more advanced learners, have students read or write a story or poem and then create a corresponding bar graph.

Reflections
Overall, our students were both engaged and successful throughout this investigation. Rich mathematical discussions occurred during the five days because of the open-ended nature of many of the activities, which required students to constantly explain and question.

Allowing students to create their own descriptive sentences not only resulted in sentences of
varying degrees of sophistication that prompted further discussions but also allowed students at all levels to succeed. As the week progressed, we noticed that the children began to move fluidly between qualitative and quantitative graphs. In particular, many of them benefited from having a “real” context from a song, a story, or a poem. They often referred to the context in their discussions of the bar graphs or to explain their reasoning.

Finally, our experience suggests that mathematical investigations using stories, poems, and songs can also contribute to improving reading comprehension and attention to detail, as well as strengthening the connection between reading and mathematics.

REFERENCES


Sayre, April Pulley, and Jeff Sayre. One is a Snail, Ten is a Crab. Cambridge, MA: Candlewick Press, 2003.

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**Investigations activity sheet 1**

My bar graph tells a story

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**Investigations activity sheet 2**

My bar graph tells a story

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**Investigations activity sheet 3**

My bar graph tells a story

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My bar graph tells a story

The unfinished graphs below are almost identical.

a) “Five Little Ducks”

b) Students explained clearly how they knew which of the two graphs went with which song: “The five little ducks came back at the end, but the speckled frogs did not.”

Students were intrigued that the same graph matches both the “Twelve Days of Christmas” song and the Twelve Days of Summer book (O’Donnell 1991). They wondered if they could “sing the book” and were excited to discover that the book’s text matches the music.
Graphs for counting stories, songs, and poems typically increase by ones, but they might also increase by other units, decrease, both increase and decrease, or change units during the story. *Leaping Lizards* (2005) by Stuart J. Murphy can be graphed in the following way.

This graph could represent *Arctic Fives Arrive* (1996) by Elinor J. Pinczes.

April Pulley Sayre’s and Jeff Sayre’s storybook *One Is a Snail, Ten Is a Crab* (2003) might be represented by this graph.
My bar graph tells a story

My cube graph

Blue

Green

Yellow
My bar graph tells a story

My mystery cube graph

Blue  Green  Red  Yellow
My bar graph tells a story

Solve the mystery graphs

Find the number of cubes for the graph drawn by ________________________________

Blue: _________  Green: _________

Red: _________  Yellow: _________