Characteristics of Shapes

Problem

Jennifer cut a 4-inch-by-4-inch square from a piece of paper. She then folded the square in half along the diagonal to form a triangle (step 1). What are some characteristics of the resulting triangle? Jennifer then folded the vertices at the two 45-degree angles of the triangle down to meet the vertex at the right angle of the triangle (step 2). The result was a square.

Take a square piece of paper the same size as Jennifer’s and repeat the folds that she made. Unfold the paper. Now, try to see what shapes you can make by refolding the paper—but only along the crease lines (step 3). As you make a shape, fill in the chart (table 1) to show that shape’s attributes. How many of the shapes are similar? How many of the shapes are nonsimilar? What relationships do you see between the number of sides and the other attributes, or characteristics, of the shapes? Can you create a 5-sided figure? Why or why not?

<table>
<thead>
<tr>
<th>Sketch of the Shape</th>
<th>Name(s) for the Shape</th>
<th>Number of Sides</th>
<th>Other Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Challenge

Add columns to the table to record other specific characteristics, such as the following:

• area of the shape
• perimeter of the shape
• types of angles contained in the shape
• number of pairs of parallel sides
The goal of the “Problem Solvers” department is to foster improved communication among teachers by posing one problem each month for teachers of grades K–6 to try with their students. Every teacher can become an author: Pose the problem to your students, reflect on your students’ work, analyze the classroom dialogue, and submit the resulting insights to this department. Through contributions to the journal, every teacher can help us all better understand children’s capabilities and thinking about mathematics. Remember that even students’ misconceptions provide valuable information.

Classroom Setup
Some students may need guidance in following the instructions. It is important that students accurately repeat the same folds that Jennifer made. In addition, when making new shapes, they should use only the crease lines that result from the initial folds. Allow your students to work with a partner or in small groups. Encourage them to find as many appropriate names as possible for each shape and to use their own words as well as formal geometric terms to describe each shape. Collect actual students’ work, make notes about interactions and discussions that took place, and document the variety of students’ solutions that you observed.

As you reflect on your experience with the problem, keep in mind the following questions:
• What difficulties did the students have in solving the problem?
• Were you surprised by any students’ responses or interpretations?
• What language did the students use to name and describe the shapes?
• Did the students relate this problem to any others that they have investigated?
• What extensions to this problem did you or your students pose?
• What did your students learn from investigating this problem?

Where’s the Math?
This problem first requires students to follow instructions for folding a square piece of paper and understand the geometric terminology of the instructions. Second, the problem requires students to create shapes by folding paper on existing creases, a task that helps develop their spatial visualization skills. Creating a table helps students organize and consolidate their mathematical thinking through communication and representation. In naming the shapes, students practice using their mathematical vocabulary. When they describe the characteristics of shapes, they have to use their mathematical vocabulary as well as think about mathematical relationships among shapes and among parts of shapes.

Share Your Students’ Work
We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please send us your thoughts and reflections. Include information about how you posed the problem, samples of your students’ work, and even photographs showing your problem solvers in action. Send your results with your name, grade level, and school by July 1, 2007, to Sheryl Stump, Department of Mathematical Sciences, Ball State University, Muncie, IN 47306-0490. Selected submissions will be published in a subsequent issue of Teaching Children Mathematics and acknowledged by name, grade level, and school.

(Solutions to a previous problem begin on the next page)

Additional Resource
Did you know that NCTM has published a collection of selected “Problem Solvers” columns?


Visit nctm.org/catalog for more on this and other NCTM resources, including professional development offerings, other publications, and online resources.
Solutions to the Dicey Situation Problem

The problem appearing in the May 2006 “Problem Solvers” section, which focuses on probability, was stated as follows:

Kristin loved playing this game with the strange dice because she would usually win. Here’s how to play this game. You need two players. Each of you constructs one of the dice following the schema pictured below. Each player rolls his or her die. The person who rolls the higher number wins that round. After 12 rounds, the person who wins the most rounds wins the game. Can you figure out Kristin’s strategy for winning? Which of the dice would you choose to win the game? Would you prefer to choose a die first or second? Explain your strategy.

James Rathbun presented the Dicey Situation problem to his third-grade class at William C. Abney Academy in Grand Rapids, Michigan. Rathbun explained the rules of the game and asked the students to predict a winning strategy and then state their reasoning in writing. Students’ predictions varied, as did their reasoning. One student wrote, “I picked the 2-2-2-2-6-6 cube [cube B] because when I added the numbers up, it came to 20, more than the others.” Another student explained, “I picked the 1-5-1-1-5-5 cube [cube D] because there were more 5s [on it] than [there were] 6s [on] the 2-2-2-2-6-6 cube,” and another student added, “I picked the 3-3-3-3-3-3 cube [cube C] because there were no low numbers on any side.”

After the students made their predictions, Rathbun had them test their theories. He had prepared templates of each die. The students cut out the templates, folded them, and taped them to make the die. Working in pairs, the students were to roll the die, collect data, and test their predictions. Rathbun recalled: “There was some confusion about rolling and tallying, so I reviewed that aspect with the class. Students commented that some of their cubes were squashed and tended to show the same side more often.” This was a good observation and helped reinforce the idea that the dice needed to be fair if the class as a whole was to make conclusions based on the data. “We finished the lesson with a discussion on lotteries and gambling and the fairness or odds,” wrote Rathbun.

Kayana Hoagland, who teaches at South Puget Sound Community College in Olympia, Washington, also regularly volunteers at Lincoln Options Elementary School to work with the second and third graders on problem solving in a math lab setting. She made sets of the four dice for her students there and created a lab sheet to help them organize their thinking (see fig. 1). She reported:

I had students work in pairs and start with dice A and D. They rolled the dice 36 times and tallied the data for 36 trials. They also filled in the chart for the theoretical probability of the two dice and compared the theoretical probability to the data collected. I chose 36 trials because it would directly correspond to the theoretical probability.
**Sample recording sheet for the Dicey Situation problem**

**Part I**
You need two players. Each of you constructs one of the dice following the schema pictured to the right. Each player rolls his or her die. The person who rolls the higher number wins that round. After 36 rounds, the person who wins the most rounds wins the game.

1. Which die will win, and why do you think so?

2. Play 36 rounds and tally the wins for each die.

<table>
<thead>
<tr>
<th>Win Tally</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Die A</td>
<td>Die D</td>
</tr>
</tbody>
</table>

3. Use your results to compute the experimental probability:
   - Probability that die A wins: \( \frac{\text{die A wins}}{36} \)
   - Probability that die D wins: \( \frac{\text{die D wins}}{36} \)

4. Complete the chart to determine the theoretical probability of each die winning.

5. What is the theoretical probability of winning with die A? With die D? Does the theoretical probability match the experimental probability that you determined from your data?

**Part II**
Now, you may choose to keep your die or trade it for any of the 4 dice shown to the right:

Record the die that each person chooses.
Partners should not choose the same die as each other.

6. Play 36 rounds and tally the wins for each die.

<table>
<thead>
<tr>
<th>Win Tally</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Die __</td>
<td>Die __</td>
</tr>
</tbody>
</table>

7. Use your results to compute the experimental probability:
   - Probability that die __ wins: \( \frac{\text{die __ wins}}{36} \)
   - Probability that die __ wins: \( \frac{\text{die __ wins}}{36} \)

8. Complete the chart to determine the theoretical probability of each die winning.

In a “fair game,” each player should have an equal chance, or probability, of winning. Based on your experiment, is this a fair game? Why or why not? Which of the dice would you choose to win the game?
Second and third graders have limited proportional reasoning skills and/or decimal equivalency knowledge so this seemed like the most accessible way for them to compare probabilities.

When asked to predict which die would help a player win more often, some students added the numbers on each die and predicted that the die with the larger sum would. It was good thinking, but the strategy failed when those students played using the B and C dice against each other; in such instances, the sum on die B is greater, but die C has the greater chance of helping a student win. “We discussed how this was possible,” wrote Hoagland. “Some students wanted to create their own die or use a standard die. I allowed this for some students. They then encountered the problem of a ‘tie,’ which meant they had to decide what to do with that situation. I had them write down what they decided as a footnote.”

Cynthia Hockman-Chupp, a teacher, freelance writer, and curriculum consultant in Canby, Oregon, presented the Dicey Situation problem to home-schooled students aged 10 and 11:

They had just completed a unit on probability so they approached this problem with a great deal of excitement. After hearing the problem, students considered which die (of the four) to choose. Their selection strategies varied. After the first person in each group chose a die, the selection process intensified. The strategy suddenly turned from choosing the best die to choosing the die that “will beat my partner.” They began to make direct comparisons between their partner’s die and the remaining options. One student wanted the 6s on die B in order to counter the 5s on her partner’s die. Another looked for a die with more [large] numbers than his partner’s.

Before starting the game, Hockman-Chupp asked, “Do you think it’s best to choose a die first or second?”

The majority [of students were] happy with the order they were assigned. All but one of those who went first thought they had the advantage. The lone dissenter didn’t think the order would affect the game. Seventy-five percent of those who went second thought they had the advantage. What an optimistic group!

Hockman-Chupp observed that as the students began to play (see fig. 2), they quickly realized that things were not necessarily as they appeared:

Most of the students who’d drawn first and picked die D lost. Those who’d picked B (and chose second) were thrilled. This led us to a discussion of how their predictions differed from their experience. One student reflected, “I went first and thought that was better. But now I see the wisdom of going second because you can counter what the other person chooses.”
I used this opportunity to review terms that we’d used in their probability unit. In a previous lesson, students had used tree diagrams to organize their information to determine theoretical probabilities. Now they used this method to demonstrate the possible combinations of rolling two of these dice (see fig. 3). Students were intrigued to see the theoretical probability results that so closely matched what they’d found in their experiment.

The solutions to probability problems are often counterintuitive. This problem lends itself well to introducing a few beginning ideas about probability. Probability problems that encourage students to experiment and collect data can help build a deeper understanding of key probability concepts.

Thanks to James Rathbun and his third-grade class at William C. Abney Academy, Grand Rapids, Michigan; Kayana Hoagland and the second and third graders at Lincoln Options Elementary School, Olympia, Washington; and Cynthia Hockman-Chupp and her students in Canby, Oregon.